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ANALYSIS OF MAJOR AIR POLLUTANT OF AN INDUSTRIALAREA IN COIMBATORE CITY USING FUZZY TOPSIS

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ABSTRACT

The quality of air was determined based on National Ambient Air Quality Standards (NQAAS). According to the Comprehensive Environmental Pollution Index, Kurichi Industrial cluster in Coimbatore district has been identified as one of the critically polluted area. The main objective of this paper is to analyze the major air pollutant in this Industrial Cluster. The multi-criteria decision-making method is applied for assessment of air pollutant of Kurichi in Coimbatore city.

KEYWORDS: *Multi-Criteria Decision Making, Air Pollutant, TOPSIS, Fuzzy TOPSIS, Triangular Fuzzy Numbers*

1. INTRODUCTION

The problems of Multi-Criteria Decision Making (MCDM) are intensely applied in many domains, such as Social Sciences, Medical Sciences, and Economics etc. MCDM problems are declared as Multi- Criteria Decision Analysis (MCDA) or Multi-Attribute Decision Making (MADM) [9]. In spite of their diversity, the MCDM have shared characteristic multiple objectives and multiple-criteria which usually are in conflict with each other. The decision makers have to assess or rank these alternatives according to the weights of the criteria. In the last decades, the MCDM techniques have become the main branch of operations research [5].

According to the concept of Fuzzy TOPSIS, the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) were defined and the distance of each alternative from the FPIS and FNIS were calculated. A closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distance to both fuzzy positive- ideal solution (FPIS) and fuzzy negative- ideal solution (FNIS) [2].

In the TOPSIS method, the weights of the criteria and the ratings of alternatives are known precisely and crisp values are used for the evaluation process. However, under many conditions, crisp data are not adequate for real-life decision problems. Therefore, the Fuzzy TOPSIS method is suggested, where the weights of criteria and ratings of alternatives are assessed by entropy crisp numbers to deal with the deficiency in the traditional TOPSIS[1]. SahayaSudha A and Rachel InbaJeba J adopted Fuzzy TOPSIS using Entropy Weights on Crop Selection[4]. FTOPSIS is applied to improve the supply chain process in the food industries [7].

2. PRELIMINARIES

The concept of triangular fuzzy number and some operational laws of triangular fuzzy numbers as follows:

2.1. Definition [6]

"Let X be a nonempty set. A fuzzy set $\tilde{A}$ of X is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle / x \epsilon X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1".

2.2 Definition [2]

"A fuzzy set $\tilde{A}$of the universe of discourse X is called a normal fuzzy set implying that $\exists\ x \epsilon X, \mu_{\tilde{A}}(x) = 1$".

2.3. Definition [2]

"A fuzzy set $\tilde{A}$ of the universe of discourse if and only if for all x_1, x_2 in X,

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min\ (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, where $\lambda \in [0,1]$".

2.4. Definition [6]

"A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible values has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that:

- There exist at least one $x \in R$with $\mu_{\tilde{A}}(x) = 1$.
- $\mu_{\tilde{A}}(x) = 1$is piecewise continuous".

2.5. Definition [3]

"A triangular fuzzy number $\tilde{A}$ can be defined by a trip let (a_1, a_2, a_3) shown below figure. The membership function $\mu_{\tilde{A}}$ is defined

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\ x < a_1 \\ \frac{x-a_1}{a_2-a_1} a_1 < x < a_2 \\ \frac{x-a_3}{a_2-a_3} a_2 < x < a_3 \\ 0\ x < a_1" \end{cases}$$

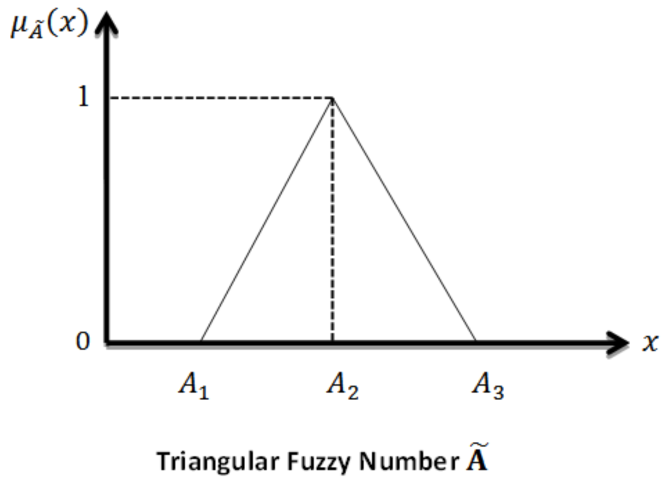

Figure 1

2.6. Definition [3]

"Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers parameterized by the triplet (a_1, a_2, a_3) and(b_1, b_2, b_3) respectively, then the operational laws of these two triangular fuzzy numbers are as follows:

$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$\tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

$$\tilde{A} \times \tilde{B} = (a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$$

$$\frac{\tilde{A}}{\tilde{B}} = (a_1, a_2, a_3)/(b_1, b_2, b_3) \;=\; (a_1/b_3, a_2/b_2, a_3/b_1)$$

$$\tilde{A} \;= (ka_1, ka_2, ka_3)\text{“}$$

2.7. Definition [3]

"Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them,

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}\text{ “.}$$

2.8. Definition [1]

"If $\tilde{A} = (a_1, a_2, a_3)$,$\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the distance of $\tilde{A}$ from $\tilde{B}$ is achieved by following relation:

$$S(\tilde{B}, \tilde{A}) = \frac{(b_1+2b_2+b_3)-(a_1+2a_2+a_3)}{4}$$

It is clear that the distance of the triangular fuzzy number $\tilde{A}$, the crisp number 0 equals following value:$S(\tilde{A}, 0) = \frac{(a_1+2a_2+a_3)}{4}$„

3. ALGORITHM OF FUZZYTOPSIS

- Form a group of decision makers.
- Evaluate the ranking of each criterion according to their significance.
- Normalize the aggregated fuzzy importance weight for each criterion.
- Form a decision matrix.
- Normalize the decision matrix.
- Construct the weighted normalized fuzzy decision matrix.
- Calculate the fuzzy positive and negative ideal solution.
- Determine the fuzzy distance of each alternative.
- Determine the fuzzy closeness coefficient and defuzzify it.
- Rank the alternatives according to their closeness coefficient.

4. THE PROPOSED STEPS OF FUZZY TOPSIS METHOD

According to this method, the best alternative would be the one that is nearest to the positive- ideal solution and farthest from the negative ideal solution. The positive-ideal solution is a solution that maximizes the profit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the profit criteria. In short, the positive-ideal solution is collected of all best values attainable from the criteria, whereas the negative ideal solution contains all worst values attainable from the criteria.

The calculation procedures of the method are as follows:

Step 1: Form a decision matrix for ranking. An MCDM problem can be briefly expressed in the matrix format.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ . \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ . & . & . & . \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nm} \end{pmatrix} \end{array}$$

where, $A_1, A_2, \dots, A_n$ are alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_m$ are criteria with alternative performance are measured, $\tilde{x}_{21} = \left(x_{ij}^a, x_{ij}^b, x_{ij}^c\right)$ is fuzzy rating of alternative, A_i with respect to criterion C_j.

Step 2: Determine the normalized decision matrix. The normalized value $\tilde{n}_{ij} = \left(n_{ij}^a, n_{ij}^b, n_{ij}^c\right)$ is calculated as:

$\tilde{n}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n}\left(s(\tilde{x}_{ij},0)\right)^2}}, j = 1,2,3,\dots,n$

where, $s(\tilde{x}_{ij},0) = \frac{x_{ij}^a + 2x_{ij}^b + x_{ij}^c}{4}$

Step 3: The weighted normalized value v_{ij} is found and the output entropy e_j of the j^{th}factor becomes

$e_j = -k\sum_{i=1}^{m} p_{ij} \ln p_{ij}, (k = \frac{1}{\ln m}, 1 \le j \le n),$

Variation coefficient of the j^{th} factor $g_j$can be defined by the following equation:

$d_j = 1 - e_j, (1 \le j \le n)$

Calculate the weight of the entropy w_j:

$w_j = \frac{g_j}{\sum_{i=1}^{m} g_j}, (1 \le j \le n)$

Step 4: The weight ednormalized value $\tilde{V}_{ij} = \left(v_{ij}^a, v_{ij}^b, v_{ij}^c\right)$is determined, considering the different important values of each criterion and, the weighted normalized fuzzy-decision matrix is constructed as, if W is a crisp value:

$\tilde{V} = \left[\tilde{V}_{ij}\right]_{n\times m}, i = 1,2,3,\dots,n; j = 1,2,3,\dots,m.$

Where, $\tilde{V}_{ij} = \tilde{x}_{ij} \times W_i, W_j$ is the weight if the i^{th} criterion, and $\sum_{j=1}^{n} W_j = 1.$

A set of performance ratings of $A_i = (i = 1,2,3,\dots,n)$ with respect to criteria

$C_j = (j = 1,2,3,\dots,m)$called $\tilde{x} = \tilde{x}_{ij}, (i = 1,2,\dots,n; j = 1,2,\dots,m)$. A set of importance weights of each criterion $W_i, i = 1,2,3,\dots,n.$

Step 5: Calculate the positive ideal solutions and negative ideal solutions respectively

$A^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+\}$

$A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\}$

Where, $\tilde{v}_j^+ = (1,1,1)$and $\tilde{v}_j^- = (0,0,0)$, j=1,2,…,m.

Step 6: Determine the separation measures using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as:

$d_i^+ = \sum_{j=1}^{m} d(\tilde{v}_{ij}, \tilde{v}_j^+), i = 1,2,\dots,n$

Similarly, the separation from the negative-ideal solution is given as

$d_i^- = \sum_{j=1}^{m} d(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1,2,\dots,n$

Where, d (.,.) is the distance between two fuzzy numbers, computed by using the n-dimensional Euclidean distance.

The separation of each alternative from the positive-ideal solution and negative ideal solution by using the equation.

$$S(\tilde{B},\tilde{A}) = \frac{(b_1+2b_2+b_3)-(a_1+2a_2+a_3)}{4}$$

Step 7: Determine the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^+ is defined as:

$$cl_i^+ = \frac{d_i^+}{d_i^+ + d_i^-}, i = 1,2,\dots,m$$

Step 8: Rank the preference order. A large value of closeness coefficient cl_i^+ indicates a good performance of the alternative A_i. The best alternative is the one with the greatest relative closeness to the ideal solution.

5. NUMERICAL EXAMPLE

In this work, the major air pollutant concentration like SO_2, NO_2, PM_{10}, $PM_{2.5}$ *in* Kurichi area of Coimbatore city was measured [8] and an attempt has been done to evaluate the highest air pollutant in three seasons, Monsoon, Summer, and Winter.

Table 1: Pollutants Emitted in Kurichi area

Pollutant	Seasons		
	Monsoon	Summer	Winter
SO_2	10.5	24.3	19.2
NO_2	12.1	15.2	18.2
PM_{10}	90.2	112.2	102.8
$PM_{2.5}$	62.2	82.3	68.3

The above data was represented by the following figure.

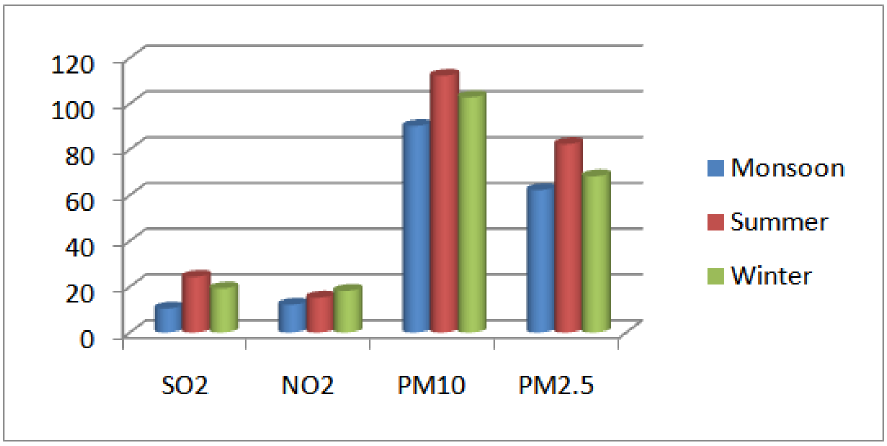

Figure 1: Pollutants Emitted in Kurichi area

The next table describes the details of the four different pollutants with three different seasons in fuzzy numbers. The alternatives $A_1, A_2, A_{3,} A_4$ are $SO_2, NO_2, PM_{10}, PM_{2.5}$ and the criteria are Monsoon, S

ummer, and Winter.

Table 2: Pollutants in Fuzzy Numbers

Alternative	Criteria		
	Monsoon	Summer	Winter
SO_2	(10.4, 10.5, 10.6)	(24.2, 24.3, 24.4)	(19.1, 19.2, 19.3)
NO_2	(12.0, 12.1, 12.2)	(15.1, 15.2, 15.3)	(18.1, 18.2, 18.3)
PM_{10}	(90.1, 90.2, 90.3)	(112.1, 112.2, 112.3)	(102.7, 102.8, 102.9)
$PM_{2.5}$	(62.1, 62.2, 62.3)	(82.2, 82.3, 82.4)	(68.2, 68.3, 68.4)

Step 1: The fuzzy decision matrix is formed from the above data.

Table 3: Fuzzy Decision Matrix

Alternative	Criteria		
	Monsoon	Summer	Winter
SO_2	(0.0939, 0.0948, 0.0957)	(0.1703, 0.1710, 0.1717)	(0.1513, 0.1521,0.1529)
NO_2	(0.1084, 0.1093, 0.1102)	(0.1063, 0.1070, 0.1077)	(0.1434, 0.1442, 0.1450)
PM_{10}	(0.8137, 0.8146, 0.8155)	(0.7891, 0.7898, 0.7905)	(0.8136, 0.8144, 0.8152)
$PM_{2.5}$	(0.5608, 0.5617, 0.5626)	(0.5786, 0.5793, 0.5800)	(0.5403, 0.5411, 0.5419)

Step 2: The procedure is as follows for the weight using Entropy analysis.

$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \ (1 \le i \le m, 1 \le j \le n),$

$\sum_{i=1}^{4} x_{i1} = 175 \ ; P_{11} = \frac{10.5}{175} = 0.06$

Similarly, $P_{21} = 0.0691$, $P_{31} = 0.5154$, $P_{41} = 0.3554$.

Table 4: Entropy Normalization Matrix

Alternative	Criteria		
	Monsoon	Summer	Winter
SO_2	0.06	0.1038	0.0921
NO_2	0.0691	0.0650	0.0873
PM_{10}	0.5154	0.4795	0.4930
$PM_{2.5}$	0.3554	0.3517	0.3276

Table 5: Weight of Criteria

Alternative	Criteria		
	Monsoon	Summer	Winter
SO_2	-0.1688	-0.2351	-0.2196
NO_2	-0.1846	-0.1777	-0.2129
PM_{10}	-0.3416	-0.3524	-0.3487
$PM_{2.5}$	-0.3677	-0.3675	-0.3656

To find the value of$P_{ij} \ln P_{ij}$, $P_{11} \ln P_{11} = 0.06 \ln 0.06$

$e_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij} \, , (k = \frac{1}{\ln m}, 1 \le j \le n), \ k = \frac{1}{\ln 4} = 0.7213$

$e_1 = -(0.7213)(-1.0627) = 0.7665$

Similarly,$e_2 = 0.8170$; $e_3 = 0.8272$

$d_1 = 1 - e_1 = 1 - 0.7665 = 0.2335$

Similarly, $d_2 = 0.1830$; $d_3 = 0.1728$

$$w_j = \frac{1-e_j}{n-\sum_{j=1}^{n} e_j}$$

$$n - \sum_{j=1}^{3} e_j = 3 - 2.4107 = 0.5893$$

$$w_1 = \frac{d_1}{0.5893} = \frac{0.2335}{0.5893} = 0.3962$$

Similarly, $w_2 = 0.3105;\ w_3 = 0.2932$

Table 6: Entropy Weight

Entropy Weight	Monsoon	Summer	Winter
$\boldsymbol{w_j}$	0.3962	0.3105	0.2932

Step 3: To find the value of $\tilde{V}_{ij} = W_j \times \tilde{n}_{ij}, i = 1,2, \dots, n; j = 1,2, \dots, m$

Table 7: Weighted Normalized Fuzzy Decision Matrix

Alternative	Criteria		
	Monsoon	Summer	Winter
$\boldsymbol{SO_2}$	(0.3962 x 0.0939=0.0372, 0.3962 x 0.0948=0.0376, 0.3962 x 0.0957=0.0379)	(0.0529, 0.0531, 0.0533)	(0.0444, 0.0446, 0.0448)
$\boldsymbol{NO_2}$	(0.0429, 0.0433, 0.0437)	(0.0330, 0.0332, 0.0334)	(0.0420, 0.0423, 0.0425)
$\boldsymbol{PM_{10}}$	(0.3224, 0.3227, 0.3231)	(0.2450, 0.2452, 0.2455)	(0.2385, 0.2388, 0.2390)
$\boldsymbol{PM_{2.5}}$	(0.2222, 0.2225, 0.2229)	(0.1797, 0.1799, 0.1801)	(0.1584, 0.1587, 0.1589)

Step 4: To find the negative and positive ideal solution:

$$S(\tilde{A}, 0) = \frac{(a_1+2a_2+a_3)}{4}$$

$$S\big((0.0372, 0.0376, 0.0379), 0\big) = \frac{0.0372+2(0.0376)+0.0379}{4} = 0.0376$$

$$S((0.0429, 0.0433, 0.0437),0) = \frac{0.0429+2(0.0433)+0.0437}{4} = 0.0433$$

$$S\big((0.3224, 0.3227, 0.3231), 0\big) = \frac{0.3224+2(0.3227)+0.3231}{4} = 0.3227$$

$$A^+ = \{(0.3224, 0.3227, 0.3231), (0.2450, 0.2452, 0.2455), (0.2385, 0.2388, 0.2390)\}$$

$$A^- = \{(0.0372, 0.0376, 0.0379), (0.0330, 0.0332, 0.0334), (0.0420, 0.0423, 0.0425)\}$$

Step 5

$$S(\tilde{B}, \tilde{A}) = \frac{(b_1+2b_2+b_3)-(a_1+2a_2+a_3)}{4}$$

$$d_1^+ = \sqrt{0.081310523 + 0.036912016 + 0.037703931} = 0.394875259$$

Similarly, $d_2^+ = 0.402051218,\ d_3^+ = 0, d_4^+ = 0.143956264$

$$d_1^- = \sqrt{0 + 0.000396010 + 0.000005406)} = 0.020035369$$

Similarly, $d_2^- = 0.005725033,\ d_3^- = 0.406050950, d_4^- = 0.263204014$

$$cl_1^+ = \frac{d_1^+}{d_1^+ + d_1^-} = 0.951711603$$

Similarly, $cl_2^+ = 0.985960357, cl_3^+ = 0, cl_4^+ = 0.353561661$

Table 8: Ranking of Alternatives

Alternative	d_i^+	d_i^-	cl_i^+	Ranking
SO_2	0.394875	0.020035	0.9517	2
NO_2	0.402051	0.005725	0.9860	**1**
PM_{10}	0	0.406051	0	4
$PM_{2.5}$	0.143956	0.263204	0.3536	3

CONCLUSIONS

From the analysis, the final ranking is based on the highest value of cl_i^+. The highest ranking has arrived in NO_2, which mean the major pollutant in Kurichi (Industrial) area is Nitrogen Dioxide. Breathing air with a high concentration of Nitrogen Dioxide can irritate airways in the human respiratory system. It aggravates respiratory diseases, particularly asthma, leading to respiratory symptoms such as coughing, wheezing, or difficulty in breathing.

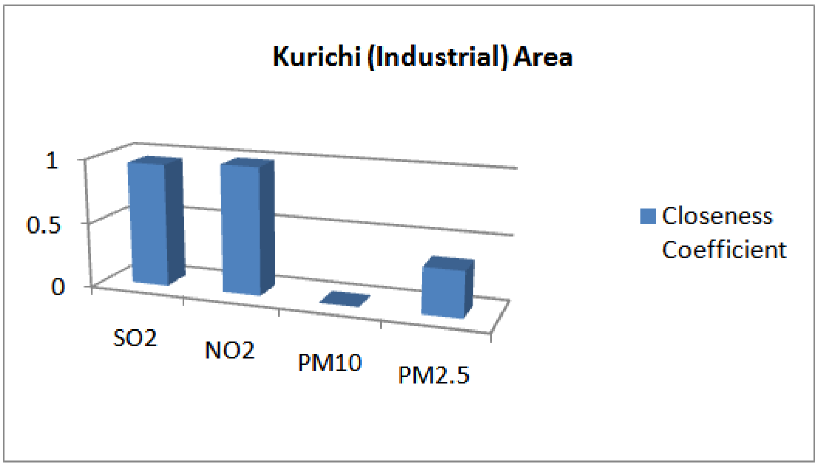

Figure 2: Ranking of Alternatives

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